An introduction to density functional theory for experimentalists

Tutorial 2.1

As usual we create a new folder on the HPC cluster:

\$ cd ~/scratch/summerschool ; mkdir tutorial-2.1 ; cd tutorial-2.1

Equilibrium structure of a diatomic molecule

In this tutorial we are going to learn how to calculate the equilibrium structures of simple systems. The formal theory required for these calculations will be discussed in Lecture 3.1, for now we can just use the following rule of thumb:

Among all possible structures, the equilibrium structure at zero temperature and zero pressure is found by minimizing the DFT total energy.

The total potential energy is the same quantity that we have been using during Tutorial 1.1 and Tutorial 1.2 (eg when we did grep "\!" silicon-1.out). This quantity includes all terms of the electron-ion Hamiltonian, except the kinetic energy of the ions.

Let us calculate the equilibrium structure of the Cl_2 molecule.

The Cl_2 molecule has only 2 atoms, and the structure is completely determined by the Cl–Cl bond length. Therefore we can determine the equilibrium structure by calculating the total energy as a function of the Cl–Cl distance.

The first step is to find a suitable pseudopotential for Cl. As in Tutorial 1.1 we go to http://www.quantum-espresso.org/pseudopotentials and look for Cl. We recognize LDA psedupotential by the label pz in the filename. Let us go for the following:

```
$ wget http://www.quantum-espresso.org/wp-content/uploads/upf_files/Cl.pz-bhs.UPF
```

We also copy the executable, job submission script, and input file from the previous tutorial:

```
$ cp ../tutorial-1.1/pw.x ./
$ cp ../tutorial-1.1/job-1.pbs ./
$ cp ../tutorial-1.1/silicon-1.in ./cl2.in
```

Now we can modify the input file in order to consider the CI_2 molecule:

```
$ more cl2.in
&control
calculation = 'scf'
prefix = 'Cl2',
pseudo_dir = './',
outdir = './'
/
```

```
&system
    ibrav = 1,
    celldm(1) = 20.0,
    nat = 2,
    ntyp = 1,
    ecutwfc = 100,
/
&electrons
    conv_thr = 1.0d-8
/
ATOMIC_SPECIES
    Cl 1.0 Cl.pz-bhs.UPF
ATOMIC_POSITIONS bohr
    Cl 0.00 0.00 0.00
    Cl 2.00 0.00 0.00
K_POINTS gamma
```

Using ibrav = 1 we select a simulation box which is simple cubic, with lattice parameter celldm(1). Here we are choosing a cubic box of side 20 bohr (1 bohr = 0.529167 Å). The keyword gamma means that we will be sampling the Brillouin zone at the Γ point, that is $\mathbf{k} = 0$. This is fine since we want to study a molecule, not an extended crystal. Note that we increased the planewaves cutoff to 100 Ry: this number was obtained from separate convergence tests.

We can now submit a job in order to check that everything will run smoothly, qsub job.pbs. In this case it is appropriate to use the submission execution flags -np 8, -npool 1. Incidentally, from the output file of this run (say cl2.out) we can see the various steps of the DFT self-consistent cycle (SCF). For example if we look for the total energy:

\$	grep "to	otal ener	rgy" cl2.	out	C	
	total	energy	. <	5 =	-55:44311765	Ry
	total	energy	\sim	=	-55.73166227	Ry
	total	energy	Qr.	= _(-55.83945219	Ry
	total	energy			-55.84156819	Ry
	total	energy		GY	-55.84161956	Ry
	total	energy	C	Q =	-55.84162104	Ry
	total	energy	A,	=	-55.84162117	Ry
!	total	energy		=	-55.84162119	Ry
	The to	otal ener	gy is the	sum of	the following	terms
		-	V7			

Here we see that the energy reaches its minimum in 8 iterations. The iterative procedure stops when the energy difference between two successive iterations is smaller than $conv_{thr} = 1.0d-8$.

Now we calculate the total energy as a function of the CI–CI bond length. In the reference frame chosen for the input file above we have one CI atom at (0,0,0) and one at (2,0,0) in units of bohr. Therefore we can vary the bond length by simply displacing the second atom along the x axis. In order to automate the procedure we can use the following script:

```
sed "s/2.00/NEW/g" cl2.in > tmp
foreach DIST ( 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0 4.2 4.4 4.6 )
   sed "s/NEW/${DIST}/g" tmp > cl2_${DIST}.in
end
```

If we create this script using copy/paste in a vi window (say vi myscript.tcsh), then we can simply issue tcsh myscript.tcsh in order to generate identical files which will differ only by the Cl-Cl bond length:

\$ ls cl2_*

cl2_2.2.in cl2_2.4.in cl2_2.6.in cl2_2.8.in cl2_3.0.in cl2_3.2.in cl2_3.4.in cl2_3.6.in cl2_3.8.in cl2_4.0.in cl2_4.2.in cl2_4.4.in cl2_4.6.in

At this point we can execute a batch job which will run pw.x for each of these input files. To this aim we simply duplicate the call to the executable inside our job-1.pbs, and modify this call in order to use the correct input file, eg:

```
mpirun -n 24 pw.x -npool 1 < cl2_2.2.in > cl2_2.2.out
mpirun -n 24 pw.x -npool 1 < cl2_2.4.in > cl2_2.4.out
...
mpirun -n 24 pw.x -npool 1 < cl2_4.4.in > cl2_4.4.out
mpirun -n 24 pw.x -npool 1 < cl2_4.6.in > cl2_4.6.out
```

After running these calculations we can look for the total energy as usual:

\$ grep "\!" cl2	_*.out	;	0	\sim	
cl2_2.2.out:!	total	energy 🔨	=	-57.51390376	Ry
cl2_2.4.out:!	total	energy	=	58.54440037	Ry
cl2_2.6.out:!	total	energy	= ~	-59.17256643	Ry
cl2_2.8.out:!	total	energy	G	-59.55021751	Ry
cl2_3.0.out:!	total	energy		-59.77201221	Ry
cl2_3.2.out:	total	energy	=	-59.89671133	Ry
cl2_3.4.out:!	total	energy	=	-59.96077584	Ry
cl2_3.6.out:!	total	energy	=	-59.98691109	Ry
cl2_3.8.out:!	total	energy	=	-59.98945934	Ry
cl2_4.0.out:!	total	energy	=	-59.97764683	Ry
cl2_4.2.out:!	total	energy	=	-59.95749243	Ry
cl2_4.4.out:!	total	energy	=	-59.93293866	Ry
cl2_4.6.out:!	total	energy	=	-59.90658486	Ry

By extracting the bond length and the energy from this data we can obtain the plot shown below:



In this plot the black dots are the calculated datapoints, and the blue line is a spline interpolation. In gnuplot this interpolation is obtained using the flag 'smooth csplines' at the end of the plot command.

By zooming in the figure we find that the bond length at the minimum is 3.725 bohr = 1.97 Å. This value is 1.5% below the measured bond length of 1.99 Å.

Binding energy of a diatomic molecule

The total energy of CI_2 at the equilibrium bond length can be used to calculate the dissociation energy of this molecule.

The dissociation energy is defined as the difference $E_{diss} = E_{Cl_2} - 2E_{Cl}$, with E_{Cl} the total energy of an isolated Cl atom.

In order to evaluate this quantity we first calculate E_{Cl_2} using the equilibrium bond length determined in the previous section. For this we modify the input file cl2.in as follows:

... ATOMIC_POSITIONS bohr Cl 0.00 0.00 0.00 Cl 3.725 0.00 0.00 ...

A calculation with this modified input file yields the total energy

 $E_{\rm Cl_2} = -59.99059545 \ {
m Ry}$

Now we consider the isolated Cl atom.

The only complication in this case is that the outermost (3p) electronic shell of Cl has one unpaired electron: $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$. In order to take this into account we can perform a **spin-polarized** calculation using the following modification of the previous input file:

```
&control
 calculation =
               'sc
 prefix = 'Cl2'
 pseudo_dir =
 outdir =
 &system
 ibrav = 1,
 celldm(1) = 20.0,
 nat = 1,
 ntyp = 1,
 ecutwfc = 100,
 nspin = 2,
 tot_magnetization = 1.0,
 occupations = 'smearing',
 degauss = 0.001,
/
&electrons
 conv_thr = 1.0d-8
```

ATOMIC_SPECIES Cl 1.0 Cl.pz-bhs.UPF ATOMIC_POSITIONS Cl 0.00 0.00 0.00 K_POINTS gamma

After running pw.x with this input file, we obtain a total energy

 $E_{\rm Cl} = -29.86386108 \ {\rm Ry}$

By combining the last two results we find

 $E_{\rm diss} = 0.262873 \ {\rm Ry} = 3.58 \ {\rm eV}$

This result should be compared to the experimental value of 2.51 eV from https://en.wikipedia. org/wiki/Bond-dissociation_energy. We can see that DFT/LDA overestimates the dissociation energy of Cl_2 by about 1 eV: interatomic bonding is slightly too strong in LDA.

Equilibrium structure of a bulk crystal

In this section we study the equilibrium structure of a bulk crystal. We consider again a silicon crystal, since we already studied the convergence parameters in Tutorial 1.2. We can make a new directory, eg

```
cd ~/scratch/summerschool/tutorial-2.1); mkdir silicon
cd silicon
```

and copy the executable, the submission script, the pseudopotential, and the input file from the folder tutorial-1.2. In this case the input file with the converged parameters for planewaves cutoff and 1 5CR001 Brillouin-zone sampling is:

```
$ more si.in
```

```
&control
 calculation = 'scf'
 prefix = 'silicon',
 pseudo_dir = './',
 outdir = './
&system
 ibrav = 2,
 celldm(1) = 1
 nat = 2,
 ntyp = 1,
 ecutwfc = 25.0,
/
&electrons
 conv_thr = 1.0d-8
/
ATOMIC_SPECIES
 Si 28.086 Si.pz-vbc.UPF
ATOMIC_POSITIONS
 Si 0.00 0.00 0.00
 Si 0.25 0.25 0.25
K_POINTS automatic
4 4 4 1 1 1
```

The **key observation** in the case of bulk crystals is that often we already have information about the structure from XRD measurements. This information simplifies drastically the calculation of the equilibrium structure.

For example, in the case of silicon, the diamond structure is uniquely determined by the lattice parameter, therefore the energy minimization is a one-dimensional optimization problem, precisely as in the case of the Cl_2 molecule.

In Tutorial 2.2 we will explore the more complicated situation where we want to decide which one among several possible crystal structures is the most stable.

To find the equilibrium lattice parameter of silicon we perform total energy calculations for a series of plausible parameters. We can generate multiple input files at once by using the following script (we can copy/paste this in a vi window: vi myscript.tcsh and then execute using tcsh myscript.tcsh):

```
sed "s/10.28/NEW/g" si.in > tmp
foreach ALAT ( 10.0 10.1 10.2 10.3 10.4 10.5 10.6 )
sed "s/NEW/${ALAT}/g" tmp > si_${ALAT}.in
end
```

Now we can execute pw.x using the generated input files. Once again we can enter all the instances of execution in the same submission script, eg:

```
mpirun -n 12 pw.x -npool 4 < si_10.0.in > si_10.0.out
...
mpirun -n 12 pw.x -npool 4 < si_10.6.in > si_10.6.out
```

After running the batch job on the cluster, we should be able to see the output files $si_10.0.out$, \cdots , $si_10.6.out$, and extract the total energies as follows:

\$ grep "\!" si_	*.out	0	
si_10.0.out:!	total energy	=	-15.84770898 Ry
si_10.1.out: 1	total energy	=	-15.85028964 Ry
si_10.2.out:!	total energy	=	-15.85121715 Ry
si_10.3.out:!	total energy 🔨	=	-15.85065982 Ry
si_10.4.out:!	total energy	=	-15.84873489 Ry
si_10.5.out:!	total energy	=	-15.84558108 Ry
si_10.6.out:!	total energy	=	-15.84131402 Ry

A plot of the total energy vs. lattice parameter is shown below:



Also in this case the black dots are the calculated datapoints, and the red line is a smooth interpolating function (obtained using 'smooth csplines' in gnuplot).

By zooming near the bottom we see that the equilibrium lattice parameter is a = 10.2094 bohr = 5.403 Å. This calculated value is very close to the measured equilibrium parameter of 5.43 Å; DFT/LDA underestimates the measured value by 0.5%.

Cohesive energy of a bulk crystal

The **cohesive energy** is defined as the heat of sublimation of a solid into its elements.

In practice the calculation is identical to the case of the dissociation energy of the CI_2 molecule: we need to take the difference between the total energy at the equilibrium lattice parameter and the total energy of each atom in isolation.

For the energy at equilibrium we just repeat one calculation using the same input files as above, this time by setting

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... celldm(1) = 10.2094, ...

This calculation yields:

 $E_{\mathrm{bulk}} =$ -15.85122170 Ry

(this value is an energy per unit cell, and each unit cell contains 2 Si atoms)

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For the isolated atom we need to consider one atom per cell, and spin-polarization as in the case of Cl. In fact the outer valence shell of silicon is 2p: \uparrow \uparrow \uparrow .

We can modify the input file as follows (this is very similar to what we have done for the CI atom, but this time the total spin is 2 Bohr magnetons)

```
&control
 calculation = 'scf
 prefix = 'silicon
 pseudo_dir =
 outdir = '
&system
 ibrav = 1,
 \operatorname{celldm}(1) = 20,
 nat = 1,
 ntyp = 1,
 ecutwfc = 25.0,
 nspin = 2,
 tot_magnetization = 2.0,
 occupations = 'smearing',
 degauss = 0.001,
&electrons
 conv_thr = 1.0d-8
```

ATOMIC_SPECIES Si 28.086 Si.pz-vbc.UPF ATOMIC_POSITIONS Si 0.00 0.00 0.00 K_POINTS gamma

The calculation for the isolated atom gives:

 $E_{\rm Si} = -7.53189352 \; {\rm Ry}$

By combining the last two results we obtain:

 $E_{\rm cohes} = E_{\rm bulk}/2 - E_{\rm Si} =$ 0.393717 Ry = 5.36 eV

The measured heat of sublimation of silicon is 4.62 eV (see pag. 71 of the Book), therefore DFT/LDA overestimates the experimental value by 16%.

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An introduction to density functional theory for experimentalists

Tutorial 2.2

Hands-on session

We create a new folder as usual:

\$ cd ~/scratch/summerschool; mkdir tutorial-2.2 ; cd tutorial-2.2

In this hands-on session we will study the equilibrium structure of simple crystals, namely **silicon** (as in Tutorial 2.1), **diamond**, and **graphite**.

Exercise 1

▶ Familiarize yourself with the steps of Tutorial 2.1, in particular:

- 1 Calculate the equilibrium lattice parameter of silicon
- 2 Calculate the cohesive energy of silicon

Exercise 2

In this exercise we will study the equilibrium structure of diamond.

The crystal structure of diamond is almost identical to the one that we used for silicon in Exercise 1. The two important differences are (i) this time we need a pseudopotential for diamond, and (ii) we expect the equilibrium lattice parameter to be considerably smaller than in silicon.

► After creating a new directory for this exercise, find a suitable pseudopotential for diamond. It is recommended to use the pseudopotential C.pz-vbc.UPF.

The link to the pseudopotential library can be found in the PDF document of Tutorial 1.1.

► Download this pseudpotential, copy over all the necessary files from tutorial-2.1, and perform a test run to make sure that everything goes smoothly.

For this test run it is sensible to use the experimental lattice parameter of diamond, 3.56 Å.

► Determine the planewaves kinetic energy cutoff ecutwfc required for this pseudopotential. You can generate the input files for various cutoff energies either manually, or by using the script on pag. 3 of Tutorial 1.2.

You should find that the total energy per atom is converged to within 10 meV when using a cutoff ecutwfc = 100 Ry.

► Determine the equilibrium lattice parameter of diamond, by performing calculations similar to those for silicon in Exercise 1.

Compare the calculated lattice parameter with the experimental value.

You should find an equilibrium lattice parameter of 6.66405 bohr = 3.5264 Å.

Using the equilibrium lattice parameter determined in the previous exercise, calculate the cohesive energy of diamond and compare your value with experiments.

For this calculation you can use the same strategy employed in Tutorial 2.1 for the cohesive energy of Si. Note that the C atom in its ground state has a valence electronic configuration $2s |\uparrow\downarrow| 2p |\uparrow$

As a reference, the cohesive energy that calculated using these settings should be around 9.08 eV (the experimental value is 7.37 eV).

▶ Plot the cohesive energy vs. volume/atom for all the lattice parameters that you considered.

The plot should look like the following.



In this exercise we study the equilibrium structure of graphite. A search for carbon allotropes in the Inorganic Crystal Structure Database (ICSD) yields the following structural information:

Home Contact Welcome to ICSDWeb. IP authenticated (135.196.210.216). Royal Society Chemistry Print Close ses										
Detailed View Entry 1 of 1										
H						Back to List View Back to Query				
Summary Collection Code 76767										
Struct.formula	С					Author	Trucano, P.; Chen, R.			
Space Group	P 63/m m c(194)									
Unit Cell	2.464(2) 2.464 6.711(4) 90. 90. 120.				Title of Article	Structure of graphite by neutron diffraction				
Cell Volume	35.29 A ³	Formula U	nits per Cell	4						
Temperature	room temperature		Pressure	atmospheric		Reference	Nature (London) (1975) 258, p136-p137			
PDF-numbers	01-089-7213 41-14	187	R-value	0.042						
Remark				💢 High Quality Data		Warnings & Comments	0 Warnings / 3 Comments			
	Event OF File	MuRacol	iloNamo	•						
	Export CIF File MyBaseFileName			Show Synoptic View		Feedback to the ICSD Editor				

From the data in this page we know that the unit cell of graphite is hexagonal, with lattice vectors

$$\mathbf{a}_1 = a \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \end{pmatrix} \mathbf{a}_2 = a \begin{pmatrix} -1/2 & \sqrt{3}/2 & 0 \\ \mathbf{a}_3 = a \begin{pmatrix} 0 & 0 & c/a \end{pmatrix}$$

(a = 2.464 Å and c/a = 2.724), and with 4 C atoms per primitive unit cell, with fractional coordinates:

$C_1:($	0	0	1/4)
$C_2:($	0	0	3/4)
$C_3:($	1/3	2/3	1/4)
$C_4:($	2/3	1/3	3/4)

► Starting from the input file that you used for diamond in Exercise 2, build an input file for calculating the total energy of graphite, using the experimental crystal structure given above.

Here you will need to pay attention to the entries ibrav and celldm() in the input. Search for these entries in the documentation page:

http://www.quantum-espresso.org/wp-content/uploads/Doc/INPUT_PW.html Here you should find the following:

Namelist: SYSTEM



Based on this information we must use ibrav = 4 and celldm(1) and celldm(3).

As a sanity check, if you run a calculation with ecutwfc = 100 and K_POINTS gamma you should obtain a total energy of -44.581847 Ry.

► A convergence test with respect to the number of **k**-points indicates that the total energy is converged to 4 meV/atom when using a shifted $6 \times 6 \times 2$ grid (6 6 2 1 1 1 with K_POINTS automatic). Using this setup for the Brillouin-zone sampling, calculate the lattice parameters of graphite a and c/a at equilibrium. Note that this will require a minimization of the total energy in a **two-dimensional** parameter space.

For this calculation it is convenient to automatically generate input files as follows, assuming that your input file is called graph.in:

- Replace the values of celldm(1) and celldm(3) by the placeholders ALAT and RATIO, respectively;
- Create a script myscript.tcsh with the following content:

```
rm tmp.pbs
foreach A ( 4.4 4.5 4.6 4.7 4.8 4.9 5.0 )
   foreach CA ( 2.50 2.55 2.60 2.65 2.70 2.75 2.80 2.85 2.90 )
    sed "s/ALAT/${A}/g" graph.in > tmp
    sed "s/RATIO/${CA}/g" tmp > graph_${A}_${CA}.in
    echo "mpirun -n 12 pw.x -npool 12 < graph_${A}_${CA}.in > graph_${A}.out" >> tmp.pbs
   end
end
```

• By running tcsh myscript.tcsh you will be able to generate input files for all these combinations of a and c/a.

The file tmp.pbs will contain all the correct execution commands, that you can copy/paste directly inside your submission script.

- Note that this will produce $7 \times 9 = 63$ input files, but the total execution time on 12 cores should be around 1 min.
- At the end you will be able to extract the total energies by using grep as usual

```
grep "\!" graph_*_*.out > mydata.txt
```

If you plot the total energies that you obtained as a function of a and c/a you should be able to get something like the following:



This plot was generated using the following commands in gnuplot (the file mydata.txt must first be cleaned up in order to obtain only three columns with the values of a, c/a, and energy):

```
set dgrid3d splines 100,100
set pm3d map
splot [] [] [:-45.599] "mydata.txt"
```

The 'splines' keyword provides a smooth interpolation between our discrete set of datapoints. The plotting range along the energy axis is restricted in order to highlight the location of the energy minimum.

Here we see that the energy minimum is very shallow along the direction of the c/a ratio, while it is very deep along the direction of the lattice parameter a. This corresponds to the intuitive notion that the bonding in graphite is very strong within the carbon planes, and very weak in between planes. By zooming in a plot like the one above you should be able to find the following equilibrium lattice parameters:

a = 2.439 Å, c/a = 2.729

From these calculations we can see that the agreement between DFT/LDA and experiments for the structure of graphite is excellent. This result is somewhat an artifact: most DFT functionals cannot correctly predict the interlayer binding in graphite due to the lack of van der Waals corrections. Since LDA generally tends to overbind (as we have seen in all examples studied so far), but it does not contain van der Waals corrections, this functional works well for graphite owing to a cancellation of errors.

For future reference let us just note that the total energy calculated using these optimized lattice parameters is -45.60104552 Ry.

Exercise 4

In this excercise we want to see how the structure of graphite that we are using in our input file looks like in a ball-and-stick model.

The software xcryden can import QE input files and visualize the atomistic structures. General info about this project can be found at http://www.xcrysden.org.

We launch xcryden by typing:

\$ xcrysden

The user interface is very simple and intuitive. The following snapshots may be helpful to get started with the visualization.





Exercise 5

▶ Which carbon allotrope is more stable at ambient conditions, diamond or graphite?

<u>Note</u>: The answer to this question is rather delicate. In nature graphite is more stable than diamond by 40 meV/atom at ambient pressure and low temperature.

Using the calculations of Exercises 2 and 3 we find that the cohesive energy of diamond is 8 meV lower than in graphite. Therefore DFT/LDA would predict diamond to be more stable, contary to experiments. This is in agreement with the following stuy by Janotti et al, http://dx.doi.org/10.1103/PhysRevB.64.174107.